Analytical Solution of Cold-air-drainage Flow Within and Above Forest Canopy

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ABSTRACT

Cold-air-drainage flow is important in pollutants transportation and ventilation for urban settlements in mountainous regions. Most of the theoretical analysis of cold-air-drainage flow is confined to bare slope, although mountains are covered by heterogeneous forest canopies. To fill this gap, a new theoretical model for cold-air-drainage flow by taking the forest canopy into account was developed in this paper. The classical Prandtl slope flow model is implemented above the canopy while the canopy flow model is applied within the canopy. The coupling of the two models is formulated at the canopy top. The predicted velocity profile is in good agreement with the field measurement data. The sensitivity analysis, including the influence of ambient stratification and slope inclination, is discussed.

Introduction

Many serious urban pollution episodes occur at the condition of weak or absent background winds. For a city located in mountainous regions, when there are no background winds, locally thermally-driven flow will play its role. During the daytime, the upslope/anabatic flow can be developed while in the night time, the flow will be downslope (katabatic) which corresponds to the heating or cooling of the slope surfaces. The downslope wind is also called cold-air-drainage flow. At this time, the pure thermally-driven slope flow is very crucial and beneficial in ventilating the urban area nearby (Kitada et al, 1998; Ohashi and Kida, 2002; Luo and Li, 2008). In addition, cold-air-drainage flow is regarded as the primary factor to influence the nocturnal ecosystem-atmosphere exchange measurements of CO_2 and other scalars in complex terrain (Lee and Hu, 2002; Turnipseed et al, 2003).

The origin of the theoretical solutions of the katabatic winds can trace back to Prandtl's model in 1942 (Prandtl, 1942). He derived a simple one-dimensional model to describe the thermally driven flow on the slope with the assumptions that the slope is infinite in length, the perturbation of the potential temperature on the slope surface is uniform along the slope, the Coriolis effect is neglected, and nonlinear advective effects are ignored. However, Prandtl's model is restricted in the case of smooth slope surface without considering the effect of the vegetative canopies. The presence of the vegetation canopies can exert resistance on the flow within it. Few authors considered the slope cover, e.g., grass, bushes and trees, on the katabatic flow. Bergen (1969) probably was the first to document cold air drainage flow within vegetation. He measured the vertical profiles of wind speed and temperature, and developed relationships between wind speed and the potential temperature drop along the slope. Yi et al (2005) proposed a model incorporating the buoyancy term in a canopy model to examine the influence of the nocturnal drainage flow on CO_2 exchanges between vegetation and atmosphere. As a matter of fact, the thermally driven slope winds not only can occur within the canopy but also can be developed above the canopy (Komatsu, et al., 2003).

In our present study, we intend to develop a theoretical model taking into account both the above and within canopy cold-air drainage flows. The coupling was formulated at the top of the canopy. The predicted vertical profile of wind velocity was compared with the on-site measurement at the Niwot Ridge AmeriFlux site located in a subalpine forest ecosystem in the Rocky Mountains of Colorado. Also, the effects of atmospheric stratification and the steepness of the slope are investigated.

Model of canopy flow

The turbulent flow in vegetative canopies has been extensively studied since the very early 1960s. Excellent reviews on this issue can be found in Finnigan (2000) and Raupach and Thom (1981). To study the spatially averaged time-mean turbulent canopy flow, double-averaging method is often applied. We consider the simplest case: one-dimensional motion in horizontally homogeneous canopies, no static pressure gradient, advection, and steady-state. As shown in Fig.1, we assume a slope with an infinite length inclined at an angle of α which everywhere has a definite excess of temperature over the stratified mass of air. The Cartesian coordinate system is set as s in the direction of along the slope surface, and n normal to the slope, the origin is located at the canopy top. The canopy height is h. The governing equations for cold-air-drainage canopy flow can be written as follows:

$$\frac{\partial \overline{u'w'}}{\partial n} = g\beta\Delta\theta\sin\alpha + c_D(n)a(n)\overline{u}^2(n) \quad (1)$$

Where, $\Delta\theta$ is the deficit of the potential temperature in the drainage flow which is assumed to be a constant. This assumption indicates a well-mixing condition within the canopy which is rather rational and widely observed in real canopies (Devito and Miller, 1983; Pypker, et al., 2007; Yi, et al., 2005). (Definitions of symbols are collected at the end of the paper.)

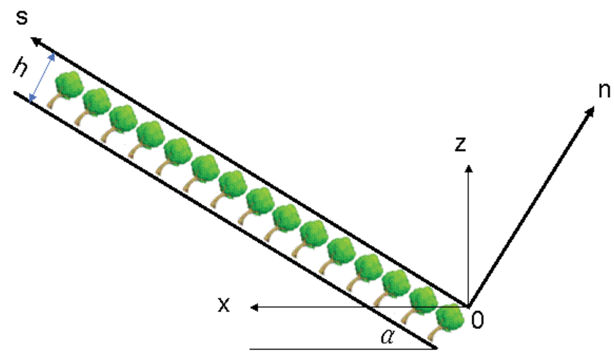
The spatial-averaged Reynolds stress $\overline{u'w'}$ can be parameterized by a simple relationship proposed by Yi (2008). He postulated a local equilibrium existing between the rate of horizontal momentum transfer and its rate of loss due to drag at an arbitrary level of n.

$$\tau(n)/\rho = -\overline{u'w'}(n) = c_D(z)\overline{u}^2(n) \quad (2)$$

Then Eq.(1) can be rewritten as

$$\frac{\partial (c_D(n)\overline{u}^2(n))}{\partial n} = g\beta\Delta\theta\sin\alpha + c_D(n)a(n)\overline{u}^2(n) \quad (3)$$

Figure 1. Schematic diagram for cold-air-drainage flow model on a forest slope



The solution to Eq.(3) can be derived, i.e.,

$$U(n) = -(\frac{c_D(0)}{c_D(n)}U^2(0)e^{-[LAI-L(n)]} - \frac{g\beta\Delta\theta\sin\alpha}{c_D(n)}\int_n^0 e^{-[L(n')-L(n)]}dn')^{1/2} \tag{4}$$

Where the cumulative leaf area per unit ground area below height *n* is defined as

$$L(n) = \int_{-h}^{n} a(n')dn' \tag{5}$$

$LAI = L(0)$ is defined as leaf area index

U(0) is the velocity at the canopy top *n* = 0 which is to be determined by coupling with the slope flow above the canopy.

Cold-air-drainage flow above the canopy

Cold-air-drainage flow above the canopy is characterized by the Prandtl model. The Prandtl model is a one dimensional model with the velocity that is expected a function of *n* only. The governing equations are shown in Eq.(6)

$$\begin{cases} g\beta\Delta\theta\sin\alpha = k_m\dfrac{d^2U(n)}{dn^2} \\ \gamma U(n)\sin\alpha = k_h\dfrac{d^2\Delta\theta}{dn^2} \end{cases} \tag{6}$$

Hence,

$$g\beta\Delta\theta\sin\alpha + \frac{k_m k_h}{\gamma\sin\alpha}\frac{d^4\Delta\theta}{dn^4} = 0 \tag{7}$$

The general solution to Eq.(10) can be obtained

$$U(n) = Ke^{-n/l}[\Delta\theta_s\sin(n/l) - C\cos(n/l)] \tag{8}$$

$$U(0) = -CK$$

Where $l = (\frac{4k_m k_h}{N^2\sin^2\alpha})^{\frac{1}{4}}$ is a mixing length, $K = \frac{g\beta}{N}\sqrt{\frac{k_h}{k_m}}$, and $\Delta\theta_s$ is the near-surface potential temperature increase from the initial background value.

There is another unknown parameter C which should be determined by coupling with the below cold-air-drainage flow as well.

Coupling at the canopy top

In order to maintain the consistency of the velocity and shear stress profile between the within- and above- canopies, the following Eqs. (9) should be satisfied.

$$\begin{cases} \bar{u}(0)_{in-canopy} = \bar{u}(0)_{above-canopy} & (9\text{-}a) \\ \Delta\theta\big|_{n=0,in-canopy} = \Delta\theta\big|_{n=0,above-canopy} = \Delta\theta_s & (9\text{-}b) \\ \dfrac{d\bar{u}}{dn}\bigg|_{n=0,in-canopy} = \dfrac{d\bar{u}}{dn}\bigg|_{n=0,above-canopy} & (9\text{-}c) \end{cases}$$

Combining Eqs. (4), (8) and (9), we can obtain that

$$C^2 - \frac{2K^2\Delta\theta_s c_D(0)}{A}C - \frac{g\beta\Delta\theta_s l\sin\alpha}{A} = 0 \tag{10}$$

$$4\Delta = b^2 - 4ac = 4\frac{K^4\Delta\theta_s^2 c_D{}^2(0) + g\beta\Delta\theta_s l\sin\alpha A}{A^2} > 0 \tag{11}$$

Therefore, $K^4\Delta\theta_s^2 c_D{}^2(0) + g\beta\Delta\theta_s l\sin\alpha A > 0$ must be satisfied in order to get the practical solution.

Here, $A = lK^2 c_D{}'(0) - lK^2 L'(0)c_D(0) - 2K^2 c_D(0)$ (12)

Hence,

$$C = \frac{K^2\Delta\theta_s c_D(0)}{A} \pm \sqrt{\Delta} \tag{13}$$

As C<0, there is only one practical solution, i.e.,

$$C = \frac{K^2\Delta\theta_s c_D(0)}{A} - \sqrt{\Delta}$$

In the end, we obtain the velocity profile for cold-air-drainage flow within and above canopies:

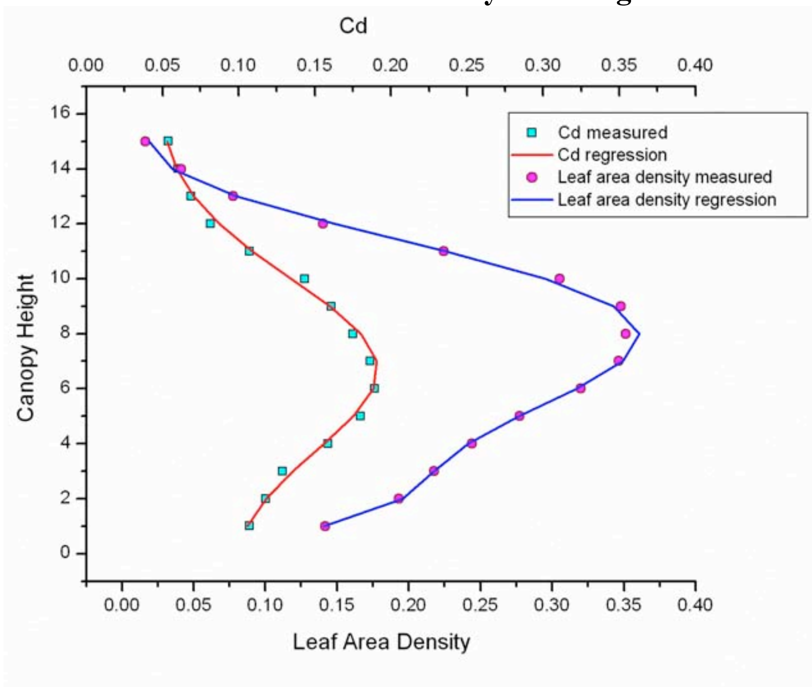
$$\begin{cases} U(n) = -\left(\dfrac{c_D(0)}{c_D(n)}K^2C^2e^{-[LAI-L(n)]} - \dfrac{g\beta\Delta\theta_s\sin\alpha}{c_D(n)}\displaystyle\int_n^0 e^{-[L(n)-L(n')]}dn'\right)^{1/2} \cdots\cdots\cdots\cdots\cdots -h \le n \le 0 \\ U(n) = Ke^{-n/l}[\Delta\theta_s\sin(n/l) - C\cos(n/l)] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots n \ge 0 \end{cases} \tag{14}$$

Validation and discussion

To validate our model, the measured velocity profile data from the Niwot Ridge forest canopy located in Rocky Mountains of Colorado, USA were used to compare with the predicted results. The terrain around the flux tower exhibits a slope of approximately 5° within 400m in the east-west direction. The forest surrounding the measurement tower is dominated by subalpine fir. The average canopy height is 15m with an average stem density of 4000 stems ha^{-1}. The canopy surface potential temperature perturbation $\Delta\theta_s$ = -2K, and the ambient atmospheric stratification

γ is assumed to be 4K/km. k_h=k_m=10 m^2/s. The velocities at different height levels including four points within canopies (1, 3, 6, 10m) and two points (21.5 and 30m) above canopies were measured. The detailed information of measurement in Niwot Ridge AmeriFlux site can be found in Turnipseed et al (2003).

Figure 2. Vertical distribution of leaf area density and drag coefficient within canopies



The leaf area density and sectional drag coefficient are shown in Fig. 2. Software Tablecurve 2.0 was used to fit the experimental data. The calculated velocity profile is compared with the measured data shown in Fig. 3. Above the canopy, the typical low-level jet profile with a maximum velocity at the lower part is observed, and within the canopy, there is a secondary maximum velocity near the ground. The minimum velocity is observed where the leaf area density reaches its maximum. Reasonable agreement between computed and measured data can be found although large discrepancy occurs at the lower part of the canopy: the trend is however quite similar. Also, the measurement points above the canopy are too limited to fully depict the jet profile, but this is the best and most suitable measurement we can find in the literature.

The effects of different atmospheric stratification conditions are shown in Fig.4. The higher ambient stratification can decrease the cold-air-drainage velocity above the canopy but little influence the flow within the canopy. Also, the strong stratification can squeeze the cold-air-drainage depth. For the steeper slope, the maximum velocities both above and within the canopy increase. However, the height at which the maximum velocity is obtained above the canopy decreases with increasing steepness.

Figure 3. Comparison of computed velocity with measurement data

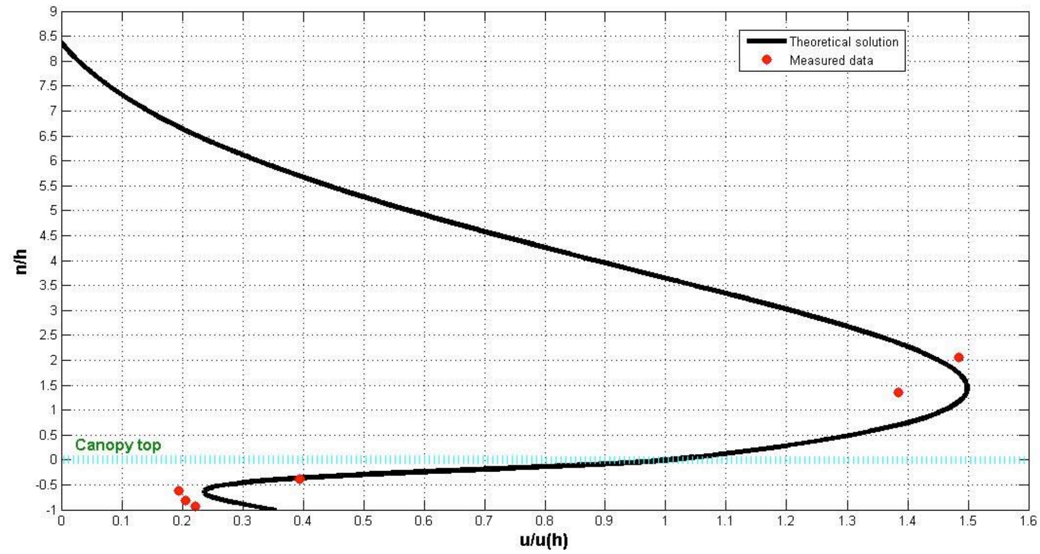
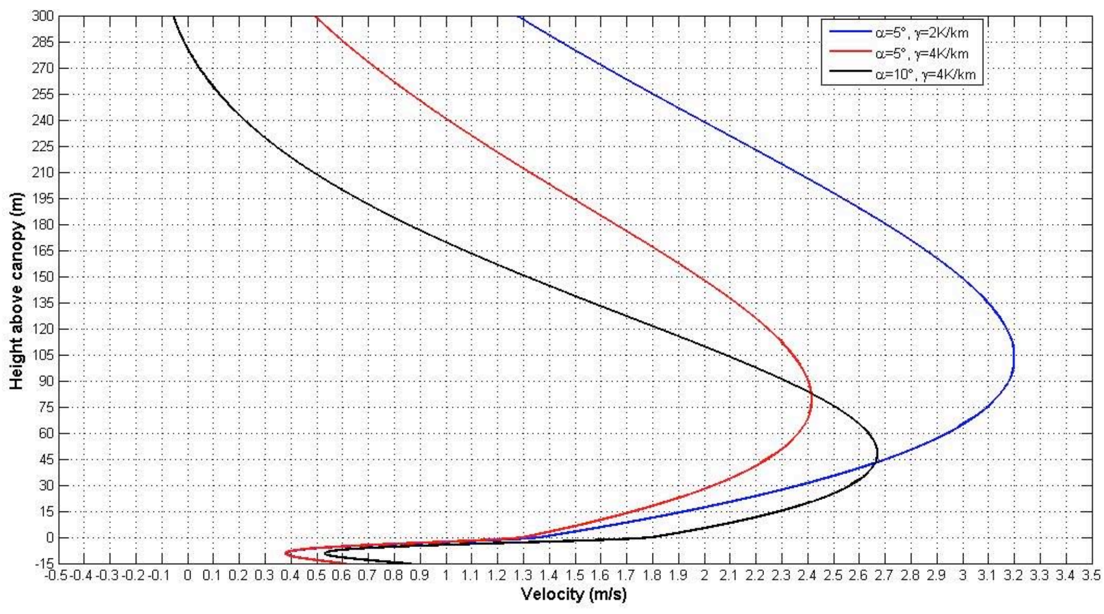


Figure 4. Effect of stratification and slope angle. The stratification is assumed to be increased from 0.002K/m to 0.004K/m, and the slope angle is increased from 5° to 10°



Conclusions

The cold-air-drainage flow is regarded as one of the effective countermeasures to UHI for a city located in the mountainous area covered by various vegetations. It is very important to know how much cold air is used to ventilate the city. The question exists: how to deal with the interaction between cold-air-drainage flow and vegetative canopies. To address this, a new theoretical model coupling both above and within cold-air-drainage flows was developed. Prandtl's model was applied to the above canopy part, while the counterpart within the canopy was described by the modified canopy flow model by taking into account the buoyancy effect. The predicted velocity profile was compared with the measured data from Niwot Ridge AmeriFlux site, and a reasonable agreement was found. Finally, the effects of ambient stratification and slope steepness were also examined. The model we propose provides a new insight on the cold-air-drainage flow on slopes covered by vegetations.

Definitions of symbols	
α	angle of the slope
$\Delta\theta$	potential temperature deficit
g	gravity acceleration
u	along slope velocity
β	thermal expansion coefficient
c_D	drag coefficient
a	leaf area density
LAI	leaf area index
u'w' bar	spatial-averaged Reynolds stress
L	cumulative leaf area per unit ground area below height n
l	mixing height
k_h	turbulent eddy viscosity
k_m	Turbulent eddy diffusivity of heat

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